# RAMAKRISHNA MISSION VIDYAMANDIRA

**(Residential Autonomous College under University of Calcutta)**

**B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2014**

**SECOND YEAR**

**MATH FOR ECO (General)**

Date : 24/12/2014

Time : 11 am – 2 pm

**Paper : III**

Full Marks : 75

**[Use a separate Answer Book for each group]**

## Group – A

**(*Answer any ten questions*)** [10×5]

1. Define directional derivative and partial derivative of a function $f: D \to \mathbb{R} (D \subseteq \mathbb{R}^2)$. Show that for the function

$$f(x,y) = \begin{cases} \dfrac{x^3 y}{x^6 + y^3} & \text{if} \quad x^2 + y^2 \neq 0 \\ 0 & \text{if} \quad x^2 + y^2 = 0 \end{cases}$$

directional derivative at (0,0) exists in every direction. [2+3]

2. Let z be a differentiable function of x and y and let $x = r\cos\theta, y = r\sin\theta$ prove that,

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2.$$ [5]

3. Consider the function

$$f(x,y) = \begin{cases} xy.\dfrac{x^2 - y^2}{x^2 + y^2} & , \quad \text{when } x^2 + y^2 \neq 0 \\ 0 & , \quad \text{when } x^2 + y^2 = 0 \end{cases}$$

check whether f(x,y) satisfies the conditions of Schwarz Theorem. [5]

4. Use Jacobians of the functions u = x + y + z, v = xy + yz + zx, w = $x^3 + y^3 + z^3 - 3xyz$ to prove that they are dependent and find relation between them. [5]

5. State and prove Euler's Theorem for two variables. [1+4]

6. Show that $f(x,y) = |x| + |y|$ has a minimum value 0 at (0,0). Is there any maximum value? [3+2]

7. Consider the function, $f: [0,1] \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1, & x \text{ is rational} \\ -1, & x \text{ is irrational} \end{cases}$$

check where f(x) is Riemann integrable? [5]

8. a) Define partition and refinement of a closed interval [a, b]. [3]
   b) Give an example of a function for which Newtonian integral does not exist but Riemann integral exist. [2]

9. Describe when the curve y = f(x) is said to be concave upwards or downwards at a given point. Find the range of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. [1+4]

10. Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters are connected by $a^2 + b^2 = c^2$, c is a constant. [5]

11. For the Gamma function '$\Gamma$' prove that $\Gamma(n+1) = n\Gamma(n)$.

    Use Beta function and Gamma function to evaluate $\int_0^{\pi/2} \sqrt{\tan x}\, dx$. [3+2]

12. Test the convergence of the following improper integral.

$$\int_0^1 \frac{dx}{x^{1/3}}.$$ [5]

13. Prove by the method of Lagrange's multiplier that of all rectangular parallelopiped of the same volume, the cube has the least surface. [5]

14. Find the points of inflexion if any of the curve $y = x(x+1)^2$. [5]

## Group – B

(***Answer <u>any five</u> questions***) [5×5]

15. Solve the differential equation :
$ayp^2 + (2x - b)p - y = 0 \ (a > 0)$. [5]

16. Find the general solution as well as singular solution of the following differential equation ; $y + px = x^4p^2$. [1+4]

17. Solve : $\frac{d^3y}{dx^3} + \frac{dy}{dx} - \frac{d^2y}{dx^2} - y = \sin 2x$. [5]

18. Solve the following differential equation by the method of variation of parameters : $\frac{d^2y}{dx^2} + a^2y = \sec ax$. [5]

19. Using the method of undetermined co-efficient solve the differential equation : $(D^2 - 4D + 4)y = e^{2x}$. [5]

20. Solve the following simultaneous linear differential equation :
$(D-1)x + Dy = 2t + 1,\ (2D+1)x + 2Dy = t$, where, $D \equiv \frac{d}{dt}$. [5]

21. Find the general solution of the differential equation : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + x$. [5]

22. Determine the Particular Integral of the differential equation: $(D^3 - 7D^2 + 16D - 12)y = \cosh 2x$. [5]

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